NPS-58Gl72071A

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THE DEVELOPMENT OF A HOMOGENEOUS NUMERICAL OCEAN MODEL FOR THE ARCTIC OCEAN

by

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July 1972

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ABSTRACT:

A numerical ocean model driven by surface stress and a source-sink distribution is developed for a homogeneous ocean. Non-linearities, lateral friction and bottom friction are included. The basin shape can be varied to accommodate a large variety of configurations. Variable bathymetry and sources/sinks around the perimeter are included. The numerical scheme is conditionally stable and has second order accuracy in space and time.

A number of test cases are run to explore the dynamic significances of the various processes represented. The possible influence of these processes on the circulation of the Arctic ocean are discussed.

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Section I - Introduction and Report Summary

The general equations that describe the flow of both the atmosphere and the ocean are well known and are available to geophysicists for the investigation of a large variety of circulation problems. For many cases of interest analytic solutions to these general equations are not possible because of significant non-linearities in the equations, or because of irregular geophysical boundary configurations, or both. Although analytic solutions are not generally available it is often possible to obtain useful approximate solutions using numerical techniques. This numerical modeling has become a powerful tool for the investigation of both the ocean and the atmosphere.

Large scale numerical models available for the description of many features of the atmosphere and large portions of the oceans can be divided into roughly two groups. The first group is exploratory in nature and is used to investigate the significance of various physical processes. The prime emphasis is to understand just why a particular system responds as it does and what effects variations in forcing have on the outcome. The results from this type modeling may, or may not, be physically realistic but in all cases should elucidate the characteristics of the flow to be expected, its sensitivity to various inputs and significant correlations that are likely to exist. The second group of numerical models is generally more advanced and is designed for forecasting geophysical fluid flow on a more or less real time basis. The relative stage of refinement and the

large amounts of high quality data required for initial conditions has inhibited the development of these type models for the ocean and essentially all of the forecasting models routinely used deal with the atmosphere.

The object of this research was to begin a numerical exploration on the large scale circulation of the Arctic Ocean.

In the past numerical models of the Arctic Ocean have concentrated their attention on the sea ice that overlies most of the Ocean (Campbell, 1965) and tried to answer questions concerning the drift and climatological permanence of the pack ice (Maykut and Untersteiner, 1971). With the exception of Campbell's use of a simplified ocean model under an ice-layer model no significant effort has been directed towards a numerical study of the actual flow of the Arctic Ocean's waters.

At the onset of this research it must be admitted that very little is actually known about Arctic Ocean dynamics and that difficulties can be anticipated in deciding on appropriate boundary conditions and input parameters for the model. In many cases field data from the Arctic is lacking, or inadequate to give the required stress fields or inflow-outflow conditions with sufficient accuracy. This then demands that best estimates of boundary conditions serve as a tentative guide and that wide ranges of parametric inputs actually be investigated. The resulting numerical exploration yields considerable insight into the relative significance of various oceanographic parameters.

In addition to the uncertainties related to the lack of actual input data the development of a new numerical model has potential difficulties

inherent in the finite difference mathematic's that must be used. Both these problems can best be addressed by the careful development of the model from simple to more complex cases in a stepwise progression and with each step being checked against any available data (from the field or analytic considerations). This model is carried through this sort of genesis.

The first step in the model to be used for the Arctic assumes homogenous water and variable depth which is in some way similar to a model used by Holland (1976). The grid system used is based on a triangular plan similar to some systems used by Williamson (1968) and Sadourny, Arakawa and Mintz (1967). Both lateral and bottom friction are included in the model. The flow is driven by stress applied at the surface (simulating the wind or ice stress) and by source-sink distributions around the edge (that simulates major channels between the Arctic and other oceans). The details of the development for this first step in the model are given in the next section. The initial check out and experimentation with this first stage in the development of an Arctic Ocean circulation model has led to some interesting results. In particular: 1) negative curl introduced into the flow results in current patterns resembling the Beaufort Gyre; and 2) the Lomonosov Ridge (Fig. 1) acts like a dynamic block which may greatly increase the significance of the circulation caused by the source-sink distribution. There is some indication from field data reported by Muench (1970) and Thorndike (1971) that the processes indicated in the model have some counterpart in actual circulation observed

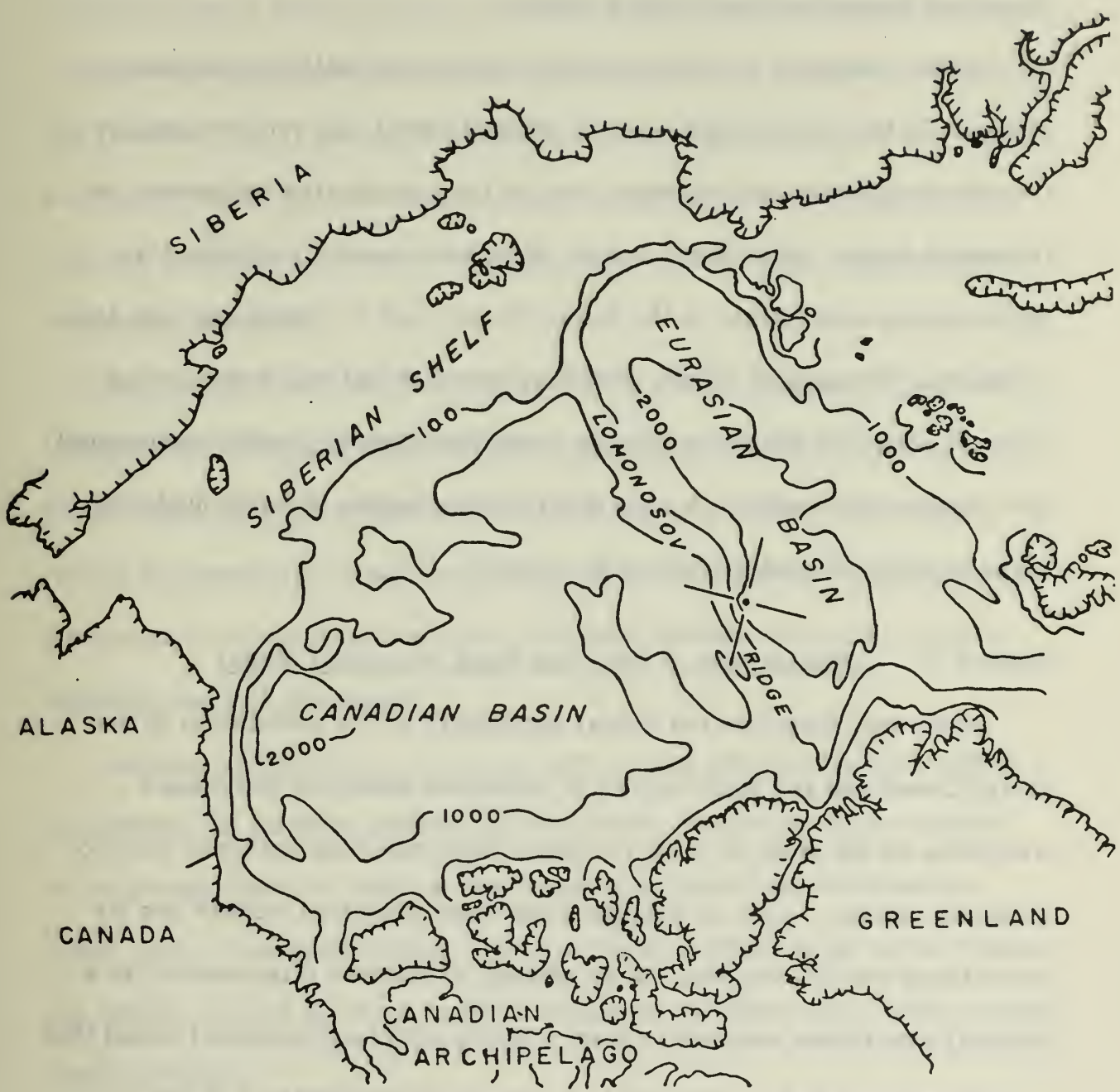

Figure 1. Chart of the Arctic Basin with the 100, 1000 and 2000 fathom depth contours drawn. Cross indicates location of the North Pole.

in the Arctic Ocean. A more detailed discussion of these results are presented in section three of this report.

Future extension and development of the model will be continued in stages. In the immediate future the present model will run with greatly increased resolution to delineate more of the complicated bathymetry of the Arctic Basin. After those results have been carefully explored the stratification appropriate to the Arctic Ocean must be introduced into the modeling. The results for the stratified ocean model will indicate how the next step (the inclusion of more complicate thermodynamic exchanges) can best be approached. A more detailed discussion of future modeling plans is given in section four of this report.

Section II - Development of the First Stage Numerical Model

When one considers the actual complexity of the circulation in the Arctic Ocean and the wide variety of numerical modeling techniques available for its study it is by no means obvious which path will lead to maximum returns. Each of the presently used numerical models has its advantages and limitations. In an attempt to address this question in a rational way it was decided to start with the simplest numerical model that could simulate what are thought to be the dominant forcing and geomorphology in the Arctic.

There can be little doubt that much of the large scale circulation of the Arctic Ocean is wind driven either directly, or indirectly through an ice cover that acts as some sort of coupling element (Campbell, 1965).

The details of how the ice cover couples the atmosphere to the ocean and to what extent it filters the time and space variations are unknown. There is however, a substantial effort directed towards this problem (Untersteiner, and Fletcher, 1971) and some progress can be anticipated. For the present model these interesting questions can not be treated in detail and thus the stress on the top of the ocean will be considered a known field, externally specified.

Studies by Coachman and Barnes (1961, 1963), Aagaard (1966) all indicate that there may be dynamically significant exchange between the Arctic Basins and the adjacent portions of the world ocean. For this reason the numerical model incoporates sources and sinks of water around its perimeter to simulate the major channels between the Arctic and the adjacent parts of the ocean.

Looking at Figure 1., it is seen that a large fraction of the Arctic is covered by the relative shallow Siberian Shelf, while the deeper portion of the Arctic Ocean is divided into two basins (both over 4000 meters deep) by the Lomonosov Ridge. In an attempt to retain some of the dynamical effects caused by these large bathymetric variations the model includes variable depth.

It is likely that the density variations in the Arctic Ocean effect the flow. For example a full treatment of the movement in the Atlantic layer (Coachman and Barnes 1963) will certainly require a baroclinic model. On the other hand there are some actual current measurements from the Arctic (Nikitin and Demyanov, 1965) (Galt, 1967) (Coachman, 1969) that indicate

a substantial barotropic, or depth independent component to the currents. This coupled with a consideration of the great increase in complexity required for variable density models suggests that for the initial numerical exploration a homogeneous or barotropic formulation be used.

The circulation in the Arctic is not well enough known to come up with an accurate appraisal of the significance of frictional forces. It seems likely that near source-sink points lateral friction could effect the flow. Over the large area covered by the Siberian Shelf it is quite possible that bottom friction might also be significant. Accordingly both lateral and bottom friction were included in the model and it was anticipated that some range of frictional parameters would be investigated to test for significance.

To develop a model with the characteristics described above the following integrated form of the equations of motion are used:

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial P}{\partial x} + K\nabla^2 u - \frac{Ru}{h} + \frac{\tau^x}{\rho h} \qquad (1)$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial P}{\partial y} + K\nabla^2 v - \frac{Rv}{h} + \frac{\tau^y}{\rho h} \qquad (2)$$

Where the dependent variables u and v are the horizontal components of velocity which are independent of depth. The density, ρ, is a constant. τ_x and τ_y, the components of the wind stress, h, the depth, and f, the coriolis parameter, are functions of position. K and R are constants that specify the effectiveness of the horizontal and vertical frictional forces respectively.

In addition to these equations of motion we have the continuity equation:

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \tag{3}$$

A transport stream function is introduced such that:

$$\begin{aligned} -hu &= \frac{\partial \psi}{\partial y} \\ hv &= \frac{\partial \psi}{\partial x} \end{aligned} \tag{4}$$

The pressure can be eliminated from equations (1) and (2) and after some manipulation the following vorticity equation is obtained:

$$\begin{aligned} &\frac{\partial \xi}{\partial \tau} - (\nabla \times \psi \vec{k}) \cdot \nabla\left(\frac{\xi + f}{h}\right) \\ &\qquad = K\nabla^2 \xi - \frac{R}{h}\left[\xi + \nabla\psi \cdot \nabla\left(\frac{1}{h}\right)\right] + \nabla \times \left(\frac{\vec{\tau}}{\rho h}\right) \end{aligned} \tag{5}$$

where:

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y}$$

$$\vec{\tau} = \tau_x \vec{i} + \tau_y \vec{j}$$

and $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit vectors in the right handed x, y, z coordinate system.

From the above the following relationship between ξ and ψ is obtained:

$$\nabla\left(\frac{1}{h}\nabla\psi\right) = \xi \tag{6}$$

Equations (5) and (6) can now be solved for the vorticity and stream

function provided that the proper initial and boundary conditions are given. In particular the following must be specified:

a) ψ - given within the region of interest at t = 0

b) ψ - given on the boundary of the region for all time

c) ξ - given within the region of interest at t = 0

Note that condition b) is equivalent to specifying the source-sink distribution around the edge of the model.

Equations (5) and (6) can be non-dimensionalized by introducing the following new variables:

$$\begin{aligned} t &= t'\left(\frac{1}{F}\right) & f &= f'F \\ x &= x'\Delta & y &= y'\Delta \\ h &= h'D & & \\ \psi &= \psi'\psi_o & & \\ \tau &= \tau'\left(\frac{\rho F \psi_o}{\Delta}\right) & & \end{aligned} \tag{7}$$

Where the primed quantities are all non-dimensional, F is the average value of the Coriolis parameter, Δ is the finite difference grid spacing, D is the average depth of the model and ψ_o is equal to the max range of stream function on the boundary of the model, or the maximum value expected for the wind driven portion of the flow, which ever is the most convenient. Using the new variables defined above equations (5) and (6) become;

$$\begin{aligned} &\frac{\partial \xi'}{\partial t} - (\nabla \times \psi' \vec{k}) \cdot \nabla\left(\frac{\alpha \xi' + f'}{h'}\right) \\ &\qquad = \beta \nabla^2 \xi' - \frac{\nu}{h'}\left[\xi' + \nabla\psi' \cdot \nabla\left(\frac{1}{h'}\right)\right] + \nabla \times \left(\frac{\vec{\tau}'}{h'}\right) \end{aligned} \tag{8}$$

$$\nabla\left(\frac{1}{h'}, \nabla\psi'\right) = \xi' \qquad (9)$$

Where:

$$\alpha = \frac{\psi_o}{\Delta^2 DF}$$

$$\beta = \frac{K}{\Delta^2 F}$$

$$\gamma = \frac{R}{DF}$$

These three non-dimensional parameters govern the character of the solution. For example setting $\alpha = 0$ removes the non-linear advective terms from the model. The size of β determines how important lateral friction is in the solution and γ scales the importance of bottom friction.

The finite difference grid that equations (8) and (9) are solved on is made up of a one dimensional array of N x M points. They are arranged as shown in figure 2. Grid points are numbered sequentially, left to right starting in the top row. This means that grid point in the neighborhood of the point L are given as shown in figure 2.

Two additional one dimensional arrays are used to specify the extent of the interior domain. These are integer arrays labeled IA and JA of dimension (M-2) and are defined so that a sweep of interior model points is obtained via the following algorithm.

```
DO 2 K=1, M-2
I = IA(K)
J = JA(K)
DO 1 L = I, J
1 Statement on Interior Point (L)
2 Continue
```

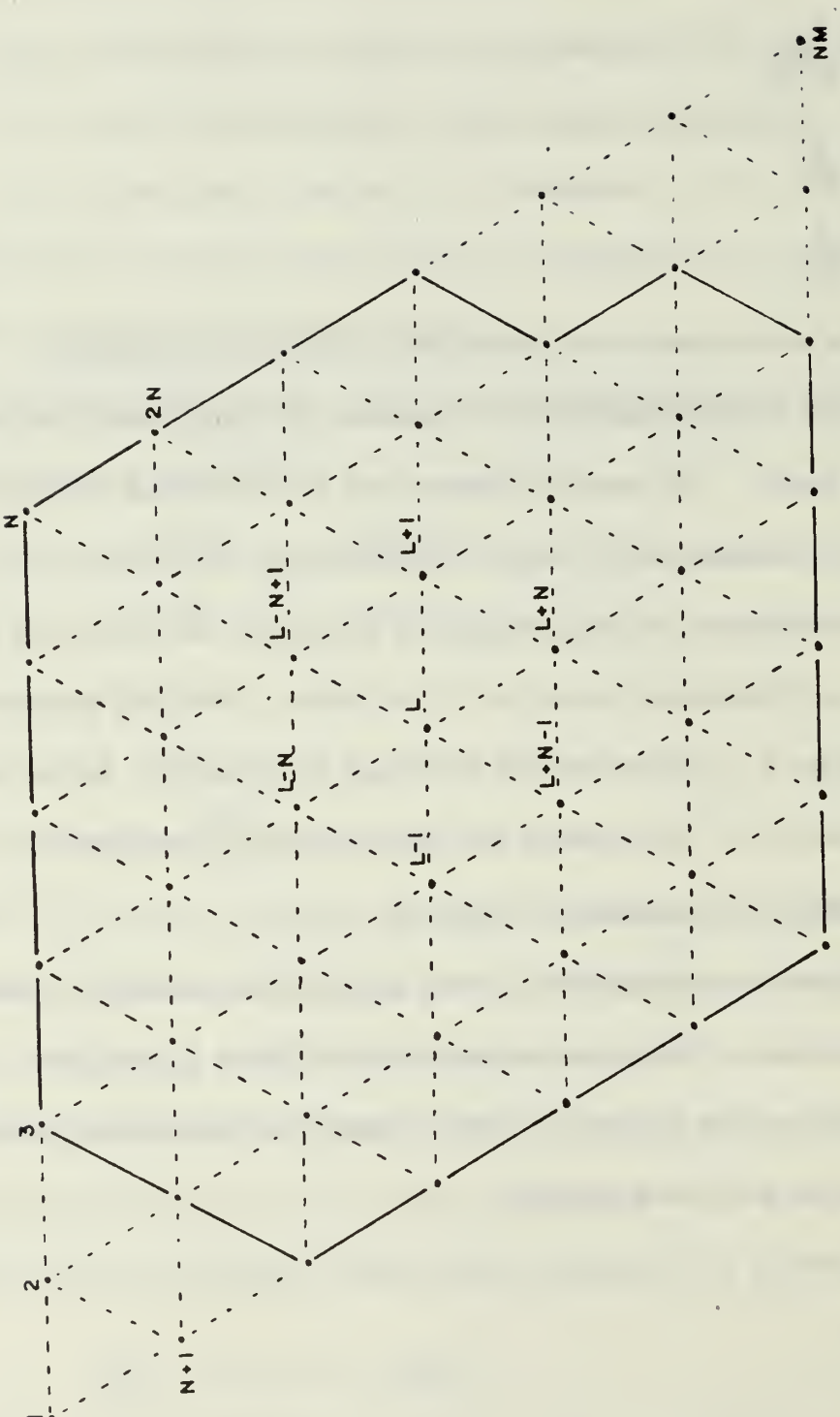

Figure 2. Grid pattern used in finite difference scheme and numbering system used for calculational molecule.

The depth in the model is specified in terms of an average depth and an array of deviations from this average. The amount of bathymetry to be used in any particular run is specified by a depth factor which is input at run time. The following depth variables are thus defined.

HM = average water depth

HFAC = depth factor

H (DIMENSIONED TO N x M) = array of depth deviations.

The actual depth used in any particular run is calculated with the following algorithm

```
      DO 3 I = 1, NM
    3 H(I) = HM + HFAC * H(I)
```

Obviously HFAC = 0 gives a constant depth model and HFAC = 1.0 includes all of the bathymetry. Any intermediate fraction is also possible if experimentation suggests such a case would be of interest.

To integrate equation (8) the three level Adams - Bashforth method is used. That is, writing equations (8) as:

$$\frac{\partial \xi'}{\partial t} = g'$$

we use:

$$\xi'(t' + \Delta t') = \xi'(t') + [\frac{3}{2} g'(t') - \frac{1}{2} g'(t'-\Delta t')]\Delta t' \qquad (10)$$

To use this equation we must write g' in finite difference form. This will be done term by term starting with g' written as:

$$g' = (\nabla \times \psi') \cdot \nabla (\frac{\alpha \xi' + f'}{h'}) + \beta \nabla^2 \xi'$$
$$- \frac{\gamma}{h'}[\xi' + \nabla \psi' \cdot \nabla(\frac{1}{h'})] + \nabla \times (\frac{\vec{\tau}}{h'}) \qquad (11)$$

The first term on the right hand side of equation (11) represents the net rate of potential vorticity advected into a unit area. To approximate this for the hexagon centered on the point L we write: (all quantities are non dimensional and primes have been omitted for simplicity)

$$(\nabla \times \psi\vec{k}) \cdot \nabla\left(\frac{\alpha\xi+f}{h}\right) = \frac{2}{3\sqrt{3}}\left[(\psi_{L-N+1} - \psi_{L+1})\left(\frac{PV_{L-N+1} + PV_{L+1}}{2}\right)\right.$$

$$+ (\psi_{L-N} - \psi_{L-N+1})\left(\frac{PV_{L-N} + PV_{L-N+1}}{2}\right) + (\psi_{L-1} - \psi_{L-N})\left(\frac{PV_{L-1} + PV_{L-N}}{2}\right)$$

$$+ (\psi_{L+N-1} - \psi_{L-1})\left(\frac{PV_{L+N-1} + PV_{L-1}}{2}\right) + (\psi_{L+N} - \psi_{L+N-1})\left(\frac{PV_{L+N} + PV_{L+N-1}}{2}\right)$$

$$\left. + (\psi_{L+1} - \psi_{L+N})\left(\frac{PV_{L+1} + PV_{L+N}}{2}\right)\right] \qquad (12)$$

Where:

$$PV_I = (C1)(\xi_I) + f_I$$

and

$$C1 = \alpha = \frac{\psi_o}{\Delta^2 DF}$$

The second term on the right hand side of equation (11) represents the diffusion of vorticity horizontally and in finite difference form becomes:

$$\beta \nabla^2 \xi = (C2)[\xi_{L+1} + \xi_{L-N+1} + \xi_{L-N} + \xi_{L-1}$$

$$+ \xi_{L+N-1} + \xi_{L+N} - (6)\xi_L] \qquad (13)$$

where

$$C2 = \frac{2\beta}{3} = \frac{2K}{3\Delta^2 F}$$

the third term on the right hand side of equation (11) represents the

dissipation of vorticity through bottom friction and is written as:

$$\frac{\gamma}{h}[\xi + \nabla\psi \cdot \nabla(\frac{1}{h})] = (\frac{C3}{h_L})\{\xi_L + (\frac{1}{4})[(\psi_{L+1} - \psi_{L-1})$$

$$+ (\frac{1}{2})(\psi_{L-N+1} - \psi_{L+N-1} - \psi_{L-N} + \psi_{L+N})]$$

$$[\frac{1}{h_{L+1}} - \frac{1}{h_{L-1}} + (\frac{1}{2})(\frac{1}{h_{L-N+1}} - \frac{1}{h_{L+N-1}} - \frac{1}{h_{L-N}} + \frac{1}{h_{L+N}})]$$

$$+ (\frac{3}{8})[\psi_{L-N+1} - \psi_{L+N-1} + \psi_{L-N} - \psi_{L+N}] \qquad (14)$$

$$[\frac{1}{h_{L-N+1}} - \frac{1}{h_{L+N-1}} + \frac{1}{h_{L-N}} - \frac{1}{h_{L+1}}]\}$$

where:

$$C3 = \gamma = \frac{R}{DF}$$

The last term on the right hand side of equation (11) is the torque added per unit time by wind stress. This is assumed constant in time and calculated only once at the beginning of the program using the following finite difference form.

$$\nabla \times (\frac{\vec{\tau}}{h}) = \frac{1}{2}\{(\frac{G2}{h})_{L+1} - (\frac{G2}{h})_{L-1}\}$$

$$- (\frac{1}{\sqrt{3}})\{(\frac{G1}{h})_{L-N+1} - (\frac{G1}{h})_{L+N} + (\frac{G1}{h})_{L-N} - (\frac{G1}{h})_{L+N-1}\} \qquad (15)$$

Where G1 is the component of the wind stress in the direction from L to L + 1 and G2 is the component of the wind stress in a direction 90 degrees to the left of G1.

Equations (12) through (15) are used in equation (11) which in turn is used in the time integration scheme defined in equation (10). Once the vorticity has been obtained for a new time step equation (9) is solved by a

successive overrelaxation technique.

The algorithm used is a slight modification of the scheme suggested by Winslow (1961). The scheme used is as follows:

A residual is calculated:

$$\begin{aligned} RES &= \nabla\left(\frac{1}{h}\nabla\psi\right) - \xi \\ &= \left(\frac{4}{3}\right)\left\{\frac{\psi_{L+1}}{(h_{L+1}+h_L)} + \frac{\psi_{L-N+1}}{(h_{L-N+1}+h_L)} + \frac{\psi_{L-N}}{(h_{L-N}+h_L)}\right. \\ &\quad \left. + \frac{\psi_{L-1}}{(h_{L-1}+h_L)} + \frac{\psi_{L+N-1}}{(h_{L+N-1}+h_L)} + \frac{\psi_{L+N}}{(h_{L+N}+h_L)} - \psi_L(HFAC_L) - \xi_L\right. \end{aligned} \tag{16}$$

where:

$$HFAC_L = \left(\frac{1}{h_{L+1}+h_L}\right) + \left(\frac{1}{h_{L-N+1}+h_L}\right) + \left(\frac{1}{h_{L-N}+h_L}\right)$$

$$\left(\frac{1}{h_{L-1}+h_L}\right) + \left(\frac{1}{h_{L+N-1}+h_L}\right) + \left(\frac{1}{h_{L+N}+h_L}\right)$$

The residual is then normalized using the coefficient of ψ_L in equation (16), i.e.,

$$d\psi = \left(\frac{4}{3}\right)\left(\frac{1}{HFAC_L}\right) RES$$

and the relaxation is then given by:

$$\psi_L = \psi_L + (R)d\psi_L \tag{17}$$

A number of tests were run to estimate the optimal value for R and over a wide range of cases a value of 1.48 was indicated. This value was then used throughout the modeling efforts reported here.

The typical experiment anticipated for the model will be to apply some

stress field to a static ocean. The model ocean will then spin-up, or develop a circulation pattern that will generally approach a steady state providing an appropriate balance of torque is possible and that the applied stress field is steady. The time dependent development of the steady state circulation and to some extent the final flow pattern will depend on the characteristics of the planetary waves that make up the transients in the model. For this reason it is of some interest to look at the propagation characteristics of these waves within the model and to estimate the errors introduced by the finite difference scheme.

To estimate how the model transients will respond a simple, free, linear Rossby wave will be considered. For this case the vorticity equation (5) reduces to:

$$\frac{\partial}{\partial \tau}\left(\frac{1}{h}\nabla^2\psi\right) = -hv \cdot \nabla\left(\frac{f}{h}\right) \tag{18}$$

A wave form of the solution is assumed and near by values used in the computational molecule (Fig. 2) taken as follows:

$$\psi_L = \psi_o \epsilon^{i(kx + \ell y - \omega t)}$$

$$\psi_{L+1} = \epsilon^{ik\Delta}\psi_L$$

$$\psi_{L-N+1} = \epsilon^{\frac{i\Delta}{2}(k+\sqrt{3}\ell)}\psi_L$$

$$\psi_{L-N} = \epsilon^{\frac{i\Delta}{2}(-k+\sqrt{3}\ell)}\psi_L$$

$$\psi_{L-1} = \epsilon^{-ik\Delta}\psi_L \tag{19}$$

$$\psi_{L+N-1} = \epsilon^{\frac{i\Delta}{2}(-k-\sqrt{3}\ell)} \psi_L$$

$$\psi_{L+N} = \epsilon^{\frac{i\Delta}{2}(k-\sqrt{3}\ell)} \psi_L$$

Where x and y refer to position in a right handed co-ordinate system superimposed on the finite difference grid with the positive x-axis in the direction form L to L + 1. It can also be assumed with no loss of generality that the x and y axes are directed east and north respectively- This then gives:

$$\frac{\partial f}{\partial x} = 0 \; ; \qquad \frac{\partial f}{\partial y} = \beta^* \tag{20}$$

To obtain a reasonable estimate of the phase velocity for the Rossby waves in the model the time differencing can be assumed exact and the depth assumed constant. Using equations (19) and (20) with the finite difference forms given in equations (12) and (13), the vorticity equation given in (18) becomes, after some simplification.

$$-i\omega \frac{4\psi_L}{3h\Delta^2} \left[\cos k\Delta + 2 \cos \left(\frac{\sqrt{3}\ell\Delta}{2}\right) \cos \left(\frac{k\Delta}{2}\right) - 3\right]$$

$$= \frac{-2i\beta^*}{\Delta} \left[\sin k\Delta + \sin \left(\frac{k\Delta}{2}\right) \cos \left(\frac{\sqrt{3}\ell\Delta}{2}\right)\right]$$

Which gives

$$\omega = \frac{\beta^*\Delta}{2} \left[\frac{\sin (k\Delta) + \sin \left(\frac{k\Delta}{2}\right) \cos \left(\frac{\sqrt{3}\ell\Delta}{2}\right)}{\cos (k\Delta) + 2 \cos \left(\frac{k\Delta}{2}\right) \cos \left(\frac{\sqrt{3}\ell\Delta}{2}\right) - 3}\right]$$

For small Δ the trig functions can be expanded using small angle approximations.

This gives:

$$\omega = -\beta^* \left[\frac{k - \frac{1}{8}(k^3 + k\ell^2)(\Delta^2)}{k^2 + \ell^2 + (\frac{k^4}{18} + \frac{k^2\ell^2}{8})(\Delta^2)} \right]$$

And from this dispersion relation it can easily be seen that the x component of the phase velocity is:

$$C_x = \frac{\omega}{k} = -\beta^* \left[\frac{1 - \frac{1}{8}(k^2+\ell^2)(\Delta^2)}{\ell^2+k^2 + (\frac{k^4}{18} + \frac{k^2\ell^2}{8})(\Delta^2)} \right] \qquad (21)$$

Clearly as Δ goes to zero this goes to the exact form

$$C_x = -\frac{\beta^*}{k^2+\ell^2}$$

and the error term is second order in Δ. This then suggests that the transient wave solutions that develop in the model will be accurately represented to second order.

The actual fortran program used for the model and an explanation of the input data required is given in an appendix.

Section III - Preliminary Results

The characteristics of the model described in the last section have been investigated in a series of tests designed to check out the numerical properties of the solutions. In a number of cases it was also possible to illustrate the physical significance that the processes represented might have on the circulation in the Arctic Ocean.

The first test was to check the part of the program that handled the

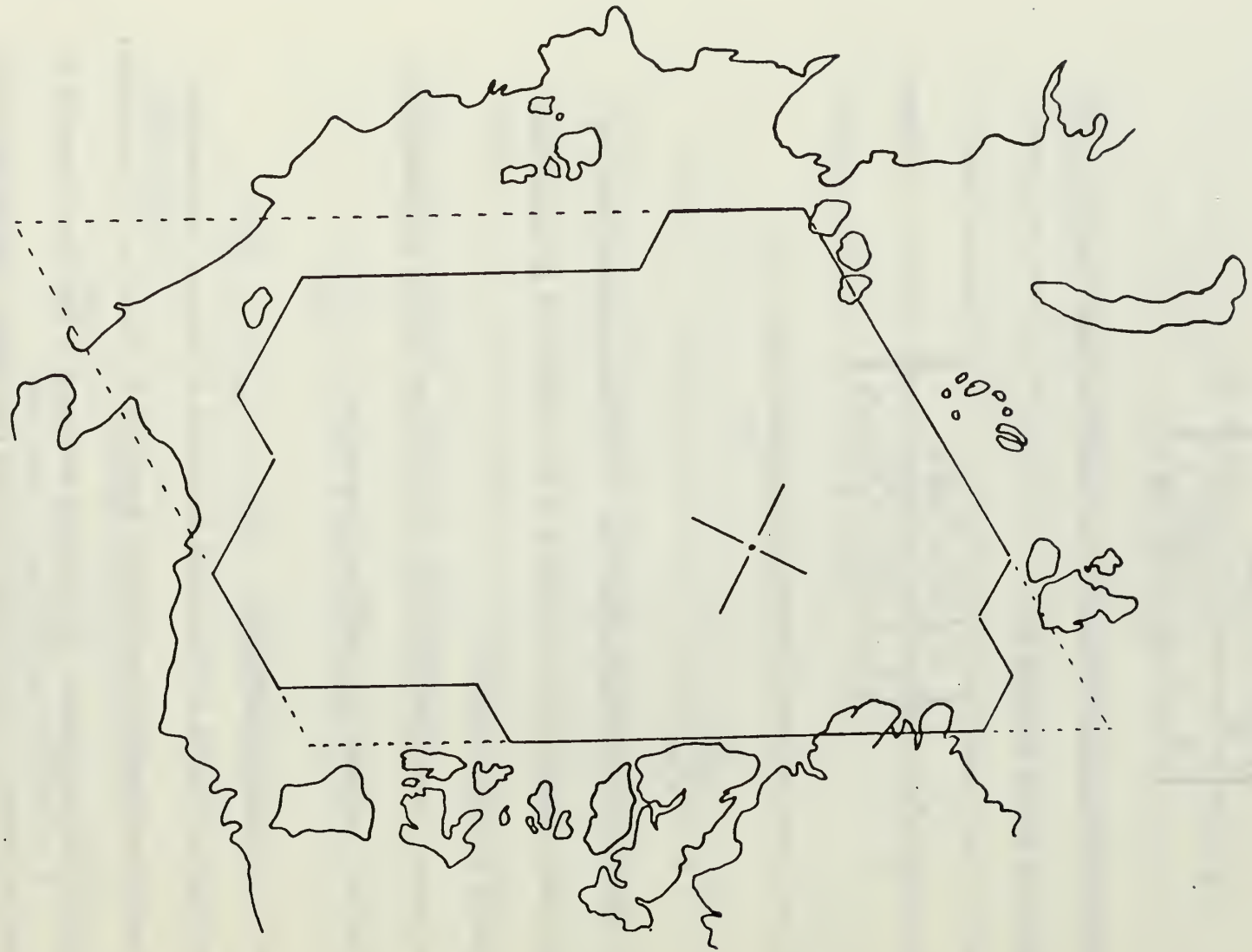

Figure 3. Configuration of the model used to check out the relaxation of the stream function.

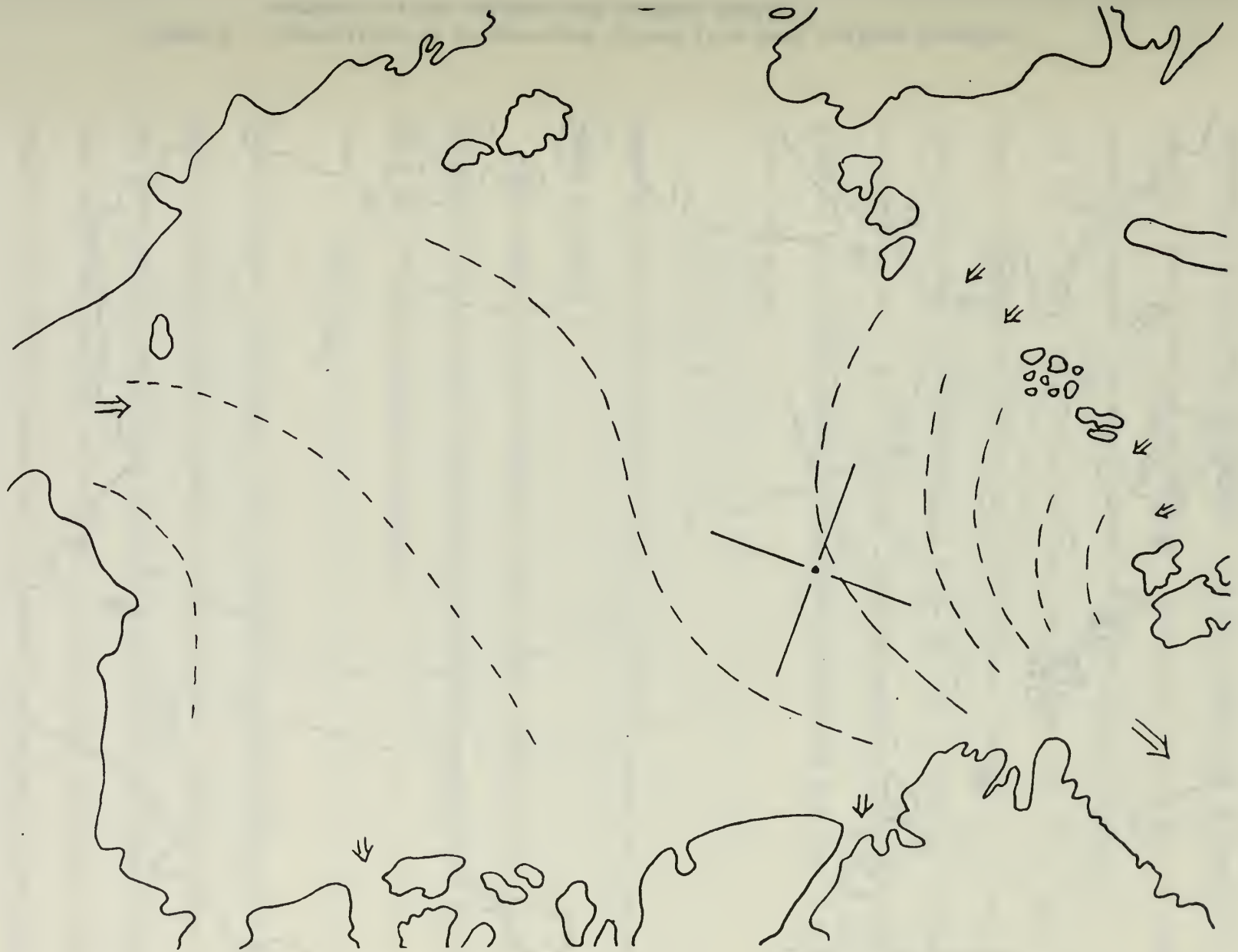

Figure 4. Streamlines of source-sink driven flow with zero vorticity in the interior and uniform depth.

Figure 5. Streamlines of source-sink driven flow with uniform negative vorticity in the interior and uniform depth.

relaxation of the stream function. A basin shape was chosen that roughly covered the deeper portion of the Arctic Ocean (Fig. 3). Representative source-sink distributions were specified as follows: 2×10^6 m^3 sec^{-1} flow in across the Chukchi Sea, 7×10^6 m^3 sec^{-1} flow in spread out on either side of Franz Josef Land, 1×10^6 m^3 sec^{-1} flow out through M'Clure Strait, 1×10^6 m^3 sec^{-1} flow out into the Lincoln Sea, and 7×10^6 m^3 sec^{-1} flow out into the East Greenland current. With a flat bottom the equations governing the flow in the interior reduce to:

$$\nabla^2 \psi = 0$$

Solving this for the specified boundary conditions gives the flow indicated in Figure 4.

The next test case introduced some finite vorticity into the above equation. For the initial testing this was simply specified as a constant. Thus the governing equation becomes:

$$\nabla^2 \psi = h\xi_o$$

This obviously corresponds to the artifical case where equation 8 has gone to a steady-uniform solution for the vorticity. Although this is not very realistic the results are interesting and shown in Figure 5 for the case where the vorticity in the flow is constant and negative as one might expect to get from the stress field developed by the polar easterlies. Circulation resembling the Beaufort Gyre develops. Qualitatively the results show a striking resemblance to the very viscous solution

for ice drift presented by Campbell (1965, Fig. 7c,d).

The next set of experiments with the model were to check out the time dependent vorticity equation (8) in conjunction with relaxation of the stream function (Eq. 9). To do this a basin roughly the shape of the Arctic was specified. Non-linear terms and bottom friction were not included ($\alpha = \gamma = 0$). A uniform negative stress curl was applied to the water that was initially at rest. The Coriolis parameter was assumed constant.

For the first series of runs made on this configuration the depth was assumed constant. Under these conditions the model would spin-up forming a symmetric clock-wise circulation pattern. The steady state solution represented a balance between the torque added by the wind and that which was lost through lateral friction. The magnitude of the steady state solution and the spin-up time of the model depended on the magnitude of the frictional coefficient (β) that was used. It is interesting to note that for this series the vorticity equation reduces to:

$$\frac{\partial \xi}{\partial t} = \beta \nabla^2 \varsigma + \frac{1}{h}(\nabla \times \tau) \tag{22}$$

With a steady wind stress this equation has no wave solutions and the appropriate von Neumann type stability condition (Richtmyer and Morton, 1967) will be of the form:

$$\delta t \leq \text{constant} \approx \frac{1}{4} \tag{23}$$

(Assuming the Adams-Bashforth scheme is used for time differencing.)

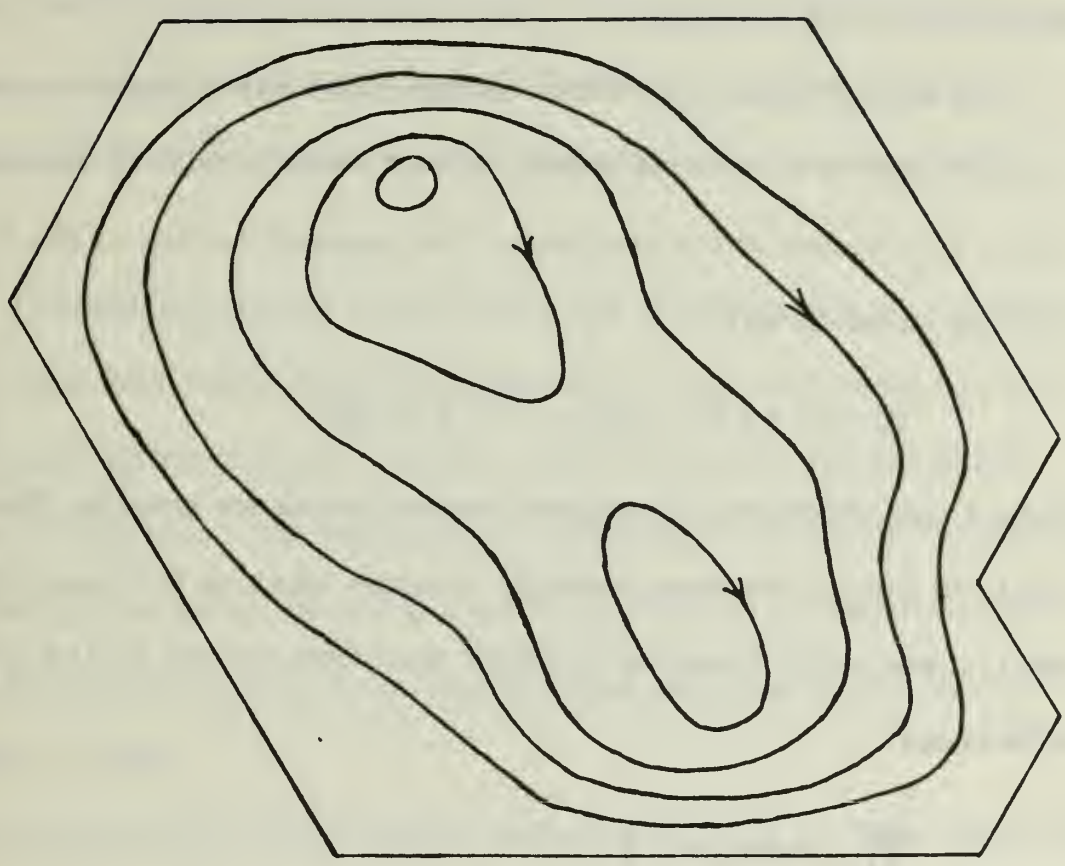

Figure 6. Streamlines of flow driven by uniform stress curl in an irregular shaped ocean with a central ridge. (The ridge runs from top to bottom in the figure.)

Where t is the non-dimensional time step in half pendulum days. This is relatively weak restriction in that with more or less realistic geophysical parameters time steps of a number of days are possible. This was confirmed with the model.

The second series run on this configuration was designed to test the models response to variable depth. In this series a ridge of smooth profile was placed across the basin. The appropriate form of the vorticity equation was

$$\frac{\partial \xi}{\partial t} - f(\nabla \times \psi \vec{k}) \cdot \nabla (\frac{1}{h}) = \beta \nabla^2 \xi + \nabla \times (\frac{\tau}{h}) \qquad (24)$$

Under these conditions topographic Rossby waves are possible (Veronis, 1966) and the von Neumann stability criterion takes on the more restrictive form: (Once again assuming an Adams-Bashforth scheme for the time differences)

$$\frac{Ct}{\Delta f} \leq \text{constant} \approx \frac{1}{2} \qquad (25)$$

Where C is the phase speed of the topographic Rossby waves. This general behavior was again confirmed by the model.

In this series the spin-up of the model begins as before, but the variable depth acts in many ways analogously to a variable F in a flat bottom ocean and western boundary type currents can develop. These stronger boundary currents develop not on the western side of the ocean basin as in Munk's model (Munk, 1950) but rather to the left of an observer looking from deep to shallow water. Figure 6 shows a typical solution

for this series. The ridge is seen to divide the flow into two cells with regions of intensified currents in each half.

During the initial spin-up for this series transient wave patterns were seen to propagate along the ridge. It is quite likely that the real time dependent stress fields applied to the Arctic ocean will result in transients of this form being propagated along the Lomonosov Ridge (Fig. 1).

An additional series of tests were run to investigate the interaction of source-sink terms with the bathymetry and the formulation used to represent bottom friction. For this series the stress applied at the surface was zero. The source-sink distribution was simply to have flow in through the Bering Straits and out through the East Greenland area. The bathymetry was again the smooth ridge described in the previous series of tests.

The first test in this series assumed $\alpha = \beta = \gamma = 0$. Thus there was no friction, or non-linear terms. The appropriate form of the vorticity equation that was:

$$\frac{\partial \xi}{\partial t} - (\nabla \times \psi \vec{k}) \cdot \nabla \left(\frac{f}{h}\right) = 0 \qquad (26)$$

This equation shows the interesting result that the only steady flow possible is when the stream lines are parallel to lines of constant f/h. Thus the steady-state flow can not cross the ridge and any source-sink distribution that demands cross ridge flow will not lead to a steady circulation. The numerical model shows these characteristics with

circulation continuing to increase with time. A strong negative (clockwise circulation builds up on the in flow side of the ridge and a corresponding positive circulation develops on the outflow side of the ridge. This is similar to the effect one might get if the water simply piled up on the in-flow side and drained out on the out-flow side satisfying the overall continuity requirements for the ocean but with minimum cross-ridge flow.

This has an important implication for the circulation in the Arctic. If the source-sink component of the flow is to be baratropic and geostrophic then flow can not cross the Lomonosov ridge. This means that continuity must be satisfied separately for both basins in the Arctic. There will be a strong tendency for flow that enters on the Canadian side of the Arctic to exit on the same side. Field data from the Arctic suggest that this may in fact be the case (Coachman, 1970). Thus one might expect that flow through the Bering Straits and the Canadian Archipelago should be strongly correllated.

The next runs in this series included the non-linear terms and lateral friction which gave the following governing vorticity equation:

$$\frac{\partial \xi}{\partial t} - (\nabla \times \psi \vec{k}) \cdot \nabla \left(\frac{\alpha \xi + f}{h}\right) = \beta \nabla^2 \xi \qquad (27)$$

For this case both frictional and non-linear boundary layers are present with cross ridge flow taking place and a steady-state solution is possible. For all reasonable values of α and β the interior of the flow is still essentially geostrophic and the cross ridge flow takes place in boundary layers

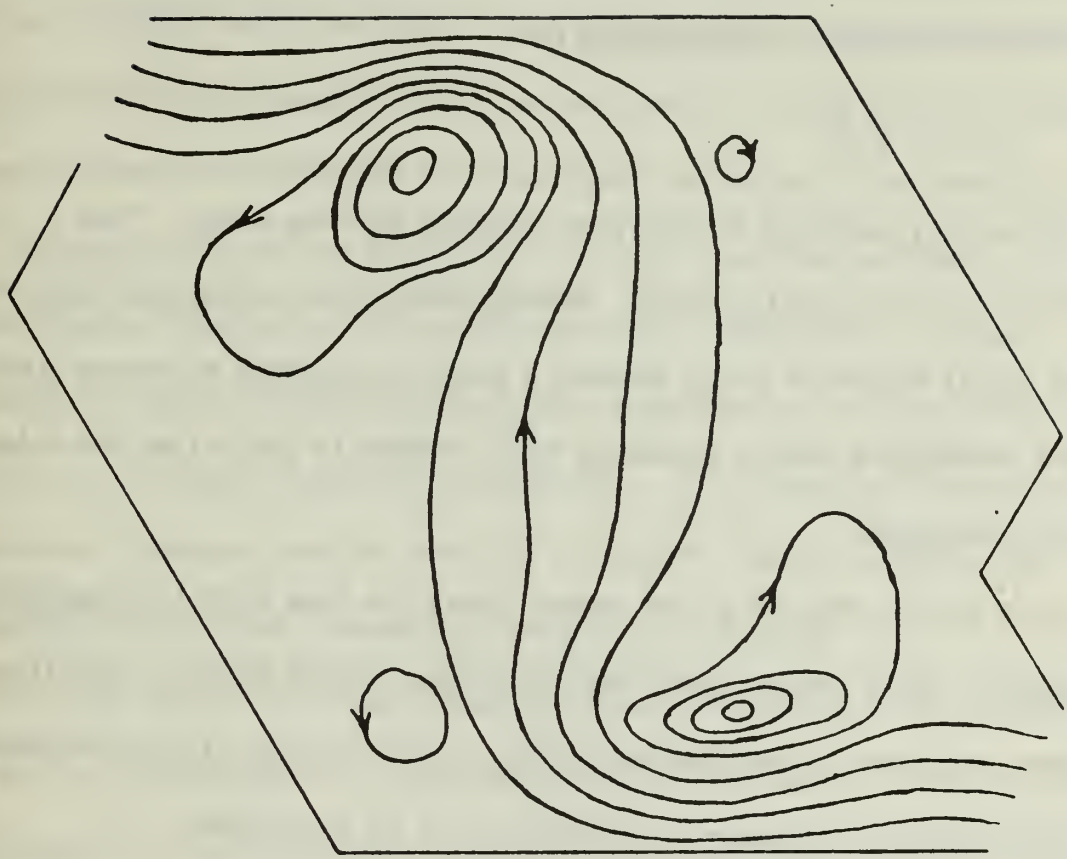

Figure 7. Streamlines of flow driven by a simple source-sink distribution for an irregular shaped ocean with a central ridge. (The ridge runs from the top to the bottom in the figure.

where the ridge intersects the sides of the basin. (Fig. 7)

A final case run in this series included the bottom friction terms. It can be anticipated that bottom friction would be effective in causing cross bathymetry flow. In particular the

$$\frac{\gamma}{h}\nabla\psi \cdot \nabla(\frac{1}{h})$$

term will have a tendency to turn flow towards shallow water. This is in fact what the model showed. Steady state flows resembled figure 7 for very small values of γ and showed a smooth transition to stream line patterns resembling simple hydraulic flow (similar to Fig. 4) as the value of γ was increased.

In the deeper interior of the Arctic Ocean the flow must be essentially geostrophic, but it seems likely that frictional effects must be significant over large portions of the Siberian shelf. In the vicinity of the Lomonosov Ridge both friction and non-linear effects may be significant.

Section IV - Future Model Plans

The next stages in the development and use of this model are well under way at the time of this writing.

The first extension is to use the model with greatly increased resolution and the actual bathymetric variation of the Arctic basins. With this configuration realistic source-sink distributions are investigated. A wind stress field obtained from Felezenbaum's pressure data (Felezenbaum, 1958) is used to drive the flow and a number of runs are

anticipated to investigate the effects of variations in α, β and γ.

The next series of tests will use the same basic model configuration but the stress field will be obtained by applying Felezenbaum's pressure data to Campbell and Rasmussen's (1971) ice model and then using the stress field from the bottom of the ice to drive the ocean model. Again a series of runs are anticipated to investigate variations in parameters.

Another series of runs is planned for the model that will have even higher spacial resolution and definitions of the bathymetry. In this series the ocean will be driven by a time dependent wind stress field that is made up of random frequency components. The dependent variables at a number of locations will be spectral analyzed with the intension of obtaining normal mode frequencies for the Arctic basins.

The next stage in the development of model exploration for the Arctic will be the formulation of a baroclinic model. The first set of experiments anticipated for this formulation will be the investigation of the dynamics associated with the circulation of the Atlantic Layer (Coachman and Barnes, 1963).

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Appendix - Use of Computer Program

The ocean model program has been written in FORTRAN - IV and a listing of the program is included in the end of this appendix. To use the program an appropriate data deck must be placed after the program. The composition of the data deck must be as follows:

1) N, M, NB FORMAT (313) - this is a single card where N and M are the dimensions of the grid system (ref. Fig. 2) and NB is the number of boundary points.

2) IA FORMAT (2013) - this is a grid number list of the first interior point of each line down the left hand side of the model.

3) JA FORMAT (2013) - this is a grid number list of the last interior point of each line down the right hand side of the model.

4) LV, BLANK FORMAT (21A1) - this is a single card with the code to be used in the graphical out put of the program. (Ref. subroutine GRAOUT)

5) HM, FAC FORMAT (2F6.1) - this is a single card where HM is the reference depth and FAC is the fraction of the bathymetry that is desired in the model.

6) H FORMAT (12F6.4) - this is a set of cards that has the normalized depth variation for each grid point.

7) F FORMAT (12F6.4) - this is a set of cards that has the normalized values of the Coriolis parameter for each grid point.

8) I FORMAT (13) - this is a single card with the exponential part of the values to be used for the wind stress.

9) GI FORMAT (12F5.3) - this is a set of cards with the significant figures part of the wind stress in the direction of the positive axis, i.e. from point L to L + 1 (Ref. Fig. 2).

10) G2 FORMAT (12F5.3) - this is a set of cards with the significant figures part of the wind stress in the direction 90^{o} to the left of the positive axis.

11) IBC FORMAT (6(6I1, I3)) - this is a set of cards that contains the information required to calculate boundary values of vorticity. For each broundary point a six digit code and the grid number are given. The six digit code refers to the neighboring points in a counter clockwise order starting with L + 1. The following code is used:
0 - exterior point, 1 - interior point (free slip B.C.), 2 - interior point (no slip B.C), 3 - boundary point along streamline, 4 - boundary point across a streamline. Boundary points at sources are not included in the IBC list.

12) C1, C2, C3, DT, TOUT, TMAX, R, CON, IPUNCH FORMAT(6E10.3, F4.2, E10.3, I2) - this is a single card that contains the run parameters. C1, C2 and C3 are defined in section two of this report. DT is the time step. TOUT is the time interval at which output of the dependent variables is desired. TMAX is the maximum time the model is to run. R is the SOR parameter to be used for the relaxation of the

stream function. CON is the absolute convergence limit to be used for the relaxation of the stream function. IPUNCH is a flag that should be non-blank if the final values for the dependent variables are required as a source deck for future runs.

13) S FORMAT (6E12.5) - this is a set of cards that contain the initial values for the stream function at each grid point.

14) V FORMAT(6E12.5) - this is a set of cards that contain the initial values for the vorticity at each grid point.

15) G1 FORMAT(6E12.6) - this is a set of cards that contain the initial values of the local time rate of chance of the vorticity at time -(-DT).

```
C     NUMERICAL OCEAN MODEL PROGRAM - DEVELOPED MAY 1971   J. A. GALT
C
C
C     ALL OF THE ARRAYS USED IN THE PROGRAM ARE DIMENSIONED HERE. IA(M2)
C     AND JA(M2) SPECIFY THE SHAPE OF THE BASIN. H(NM), F(NM) AND T(NM)
C     ARE USED FOR THE INDEPENDENT VARIABLES WATER DEPTH, CORIOLIS
C     PARAMETER AND WIND STRESS CURL RESPECTIVELY.  THE DEPENDENT
C     VARIABLES ARE ASSOCIATED WITH S(NM) FOR THE STREAM FUNCTION AND
C     V(NM) FOR THE VORTICITY.  G1(NM) AND G2(NM) ARE CALCULATIONAL
C     ARRAYS USED FOR TEMPORARY STORAGE AND LV(20) AND LO((2*N+M-2)*M)
C     ARE USED FOR THE OUTPUT OF ARRAY DATA.  IBC(7,NB) IS AN ARRAY
C     THAT CONTAINS THE COEFFICIENTS NEEDED TO CALCULATE BOUNDARY VALUES
C     FOR THE VORTICITY. RES(NM) IS AN ARRAY USED IN THE VARIABLE DEPTH
C     PART OF THE STREAM FUNCTION RELAXATION.  LA(6) IS A STORAGE ARRAY
C     USED DURING THE CALCULATION OF VORTICITY ON THE BOUNDARY POINTS.
C     HC1(NM) AND HC2(NM) ARE COEFFICIENT ARRAYS USED IN THE
C     CALCULATION OF BOTTOM STRESS.
C
      DIMENSION IA(5),JA(5),H(49),F(49),T(49),G1(49),G2(49),LV(20),
     E          LO(133),IBC(7,18),RES(49),V(49),S(49),LA(6),
     E          HC1(49), HC2(49)
C
C     THE PROGRAM STARTS BY READING IN THE SIZE OF THE MAJOR ARRAY
C     WHICH WILL BE N BY M POINTS AND THE NUMBER OF BOUNDARY
C     POINTS NB
C
      READ(5,100) N,M,NB
  100 FORMAT(3I3)
C
C     SIZE PARAMETERS ARE NOW CALCULATED.  NM IS THE TOTAL NUMBER
C     OF GRID POINTS AVAILABLE TO THE DEPENDENT VARIABLES.  NOUT
C     IS THE NUMBER OF POINTS USED IN THE OUTPUT ARRAY.  AND, M2
C     IS THE SIZE OF THE ARRAYS USED TO SPECIFY THE BASIN SHAPE.
C
      NM = N*M
      NOUT = (2*N+M-2)*M
      M2 = M-2
C
C     THE ARRAYS THAT SPECIFY THE BASIN SHAPE ARE NOW READ IN
C
      READ(5,101) IA
      READ(5,101) JA
  101 FORMAT(20I3)
C
C     THE ALPHA CODE USED IN THE OUTPUT ARRAYS IS READ IN
C
      READ(5,102) LV,BLANK
  102 FORMAT(21A1)
```

```
      WRITE(6,103)                                                      0049
  103 FORMAT(1H1,22H NUMERICAL OCEAN MODEL ///,                         0050
     E          19H THE BASIN SHAPE IS  ///)                           0051
      DO 2 K = 1,M2                                                     0052
      I = IA(K)                                                         0053
      J = JA(K)                                                         0054
      DO 1 L = I,J                                                      0055
    1 H(L) = FLOAT(L)                                                   0056
    2 CONTINUE                                                          0057
      CALL GRAOUT(LV,LO,NOUT,H,NM,IA,JA,M2,N,BLANK)                     0058
C                                                                       0059
C     THE REFERENCE DEPTH HM AND THE DEPTH FACTOR FAC ARE READ          0060
C     IN.  AFTER THIS THE NORMALIZED DEPTH VARIATIONS ARE               0061
C     READ IN AND THE WORKING DEPTHS CALCULATED.                        0062
C                                                                       0063
      READ(5,104) HM,FAC                                                0064
  104 FORMAT(2F6.1)                                                     0065
      READ(5,105) H                                                     0066
  105 FORMAT(12F5.3)                                                    0067
      DO 20 I = 1,NM                                                    0068
   20 H(I) = 1.0+FAC*H(I)                                               0069
      WRITE(6,106) HM,FAC                                               0070
  106 FORMAT(1H1,25H THE REFERENCE DEPTH IS = ,F8.2,7H METERS,//         0071
     E       22H THE DEPTH FACTOR IS =  ,F6.3//)                        0072
      WRITE(6,107)                                                      0073
  107 FORMAT(40H THE ARRAY OF NORMALIZED DEPTH VALUES IS  ///)          0074
      WRITE(6,108) H                                                    0075
  108 FORMAT(12F8.3)                                                    0076
      WRITE(6,109)                                                      0077
  109 FORMAT(// 18H BOTTOM TOPOGRAPHY //)                               0078
      CALL GRAOUT(LV,LO,NOUT,H,NM,IA,JA,M2,N,BLANK)                     0079
C                                                                       0080
C     THE NORMALIZED VALUES OF THE CORIOLIS PARAMETER ARE NOW READ IN   0081
C                                                                       0082
      READ(5,110) F                                                     0083
  110 FORMAT(12F6.4)                                                    0084
      WRITE(6,111)                                                      0085
  111 FORMAT(1H1,46H THE DISTRIBUTION OF THE CORIOLIS PARAMETER IS  ///) 0086
      WRITE(6,170) F                                                    0087
  170 FORMAT(12F8.4)                                                    0088
      CALL GRAOUT(LV,LO,NOUT,F,NM,IA,JA,M2,N,BLANK)                     0089
C                                                                       0090
C     THE PROGRAM NOW READS IN TWO COMPONENTS OF THE NORMALIZED WIND    0091
C     STRESS.  G1 GIVES ITS COMPONENT IN THE POSITIVE AXIS DIRECTION    0092
C     AND G2 GIVES ITS MAGNITUDE IN A DIRECTION 90 DEGREES TO THE LEFT  0093
C     THE STRESS HAS BEEN NONDIMENSIONALIZED USING  T =(RHO*F*S1/L)T*.   0094
C     TO SAVE SPACE ON THE INPUT CARDS THE EXPONENT PART OF THE STRESS   0095
C     IS READ IN FIRST                                                  0096
```

```
C                                                                       0097
      DO 21 I = 1,NM                                                    0098
 21   T(I) = 0.0                                                        0099
      READ(5,112) I                                                     0100
 112  FORMAT(I3)                                                        0101
      FAC = 10.0**I                                                     0102
      READ(5,105) G1                                                    0103
      READ(5,105) G2                                                    0104
      DO 3 I = 1,NM                                                     0105
      G1(I) = FAC*G1(I)/H(I)                                            0106
 3    G2(I) = FAC*G2(I)/H(I)                                            0107
C                                                                       0108
C     HERE THE CURL OF T/H IS CALCULATED                                0109
C                                                                       0110
      AREA = 2.598376                                                   0111
      FAC1 = 1.0/SQRT(3.)                                               0112
      DO 5 K = 1,M2                                                     0113
      I = IA(K)                                                         0114
      J = JA(K)                                                         0115
      DO 4 L = I,J                                                      0116
      L1 = L+1                                                          0117
      L2 = L-N+1                                                        0118
      L3 = L-N                                                          0119
      L4 = L-1                                                          0120
      L5 = L+N-1                                                        0121
      L6 = L+N                                                          0122
      T(L) = 0.5*(G2(L1)-G2(L4)-(G1(L2)-G1(L6)+G1(L3)-G1(L5))*FAC1)     0123
C                                                                       0124
C     WHILE IN THE LOOP RES(NM),HC1(NM) AND HC2(NM) ARE ALL CALCULATED  0125
C     FOR USE LATER IN THE RELAXATION OF THE STREAM FUNCTION AND THE    0126
C     BOTTOM STRESS TERM                                                0127
C                                                                       0128
      FAC = H(L)                                                        0129
      RES(L) = 1.0/(FAC+H(L1))+1.0/(FAC+H(L2))+1.0/(FAC+H(L3))          0130
     E   +1.0/(FAC+H(L4))+1.0/(FAC+H(L5))+1.0/(FAC+H(L6))               0131
      HC1(L) = 0.25 * (1./H(L1) - 1./H(L4) + 0.5 * (1./H(L2) - 1./H(L5)) 0132
     1       - 1./H(L3) + 1./H(L6)))                                    0133
      HC2(L) = (1./H(L2) - 1./H(L5) + 1./H(L3) - 1./H(L6)) * (3./8.)    0134
 4    CONTINUE                                                          0135
 5    CONTINUE                                                          0136
      WRITE(6,180)                                                      0137
 180  FORMAT(1H1,45H THE DISTRIBUTION OF THE WIND STRESS CURL IS   ///) 0138
      WRITE(6,173) T                                                    0139
      WRITE(6,118)                                                      0140
      CALL GRAOUT(LV,LO,NOUT,T,NM,IA,JA,M2,N,BLANK)                     0141
C                                                                       0142
C     THE ARRAY THAT CONTAINS THE INFORMATION NEEDED TO CALCULATE       0143
C     THE BOUNDARY VALUES FOR THE VORTICITY IS NOW READ IN.             0144
```

```
C                                                                       0145
      READ(5,113) IBC                                                   0146
  113 FORMAT(6(6I1,I3))                                                 0147
C                                                                       0148
CC    RUN PARAMETERS ARE READ IN AT THIS POINT. C1 IS THE               0149
CC    NON-LINEAR COEFFICIENT S1/FDL**2, C2 IS THE FRICTIONAL            0150
CC    COEFFICIENT 2K/3FL**2, C3 IS THE BOTTON STRESS                    0151
CC    COEFFICIENT R/DF, DT IS THE NON-DIMENSIONAL TIME                  0152
CC    STEP, TOUT IS THE INTERVAL AT WHICH DEPENDENT VARIABLES ARE       0153
CC    PRINTED OUT, TMAX IS THE MAXIMUM MODEL TIME, R IS THE             0154
CC    RELAXATION PARAMETER, CON IS THE RELATIVE CONVERGENCE FACTOR      0155
CC    FOR THE RELAXATION OF THE STREAM FUNCTION AND IPUNCH              0156
CC    IS A FLAG USED IF THE FINAL VALUE OF THE DEPENDENT                0157
CC    VARIABLES IS PUNCHED OUT AS A DATA DECK.                          0158
C                                                                       0159
      READ(5,114) C1,C2,C3,DT,TOUT,TMAX,R,CON,IPUNCH                    0160
  114 FORMAT(6E10.3,F4.2,E10.3,I2)                                      0161
      WRITE(6,171)                                                      0162
  171 FORMAT(1H1, 'THE RUN PARAMETERS C1,C2,C3,DT,TOUT,TMAX,R,CON AND   0163
     1IPUNCH ARE' ///)                                                  0164
      WRITE(6,172) C1,C2,C3,DT,TOUT,TMAX,R,CON,IPUNCH                   0165
  172 FORMAT(8E14.3, I6)                                                0166
C                                                                       0167
CC    THE INITIAL  VALUES FOR THE VORTICITY, STREAM FUNCTION AND        0168
CC    THE LOCAL CHANGE IN THE VORTICITY ONE TIME STEP BACK              0169
CC    ARE READ INTO THE PROGRAM                                         0170
C                                                                       0171
      READ(5,115) S                                                     0172
      READ(5,115) V                                                     0173
      READ(5,115) G1                                                    0174
  115 FORMAT(6E12.5)                                                    0175
C                                                                       0176
CC    THE TIME CONTROL IS STARTED NOW AND THE ACTUAL                    0177
CC    CALCULATION OF THE PROGRAM BEGINS                                 0178
C                                                                       0179
      ITER = 0                                                          0180
      TIME = 0.0                                                        0181
    6 CONTINUE                                                          0182
      FAC = TIME/TOUT+0.01*DT                                           0183
      I = IFIX(FAC)                                                     0184
      FAC = FLOAT(I)                                                    0185
      FAC = TOUT*FAC                                                    0186
      FAC = TIME-FAC                                                    0187
      FAC = ABS(FAC)                                                    0188
      DTERR = 0.1*DT                                                    0189
      IF (FAC.GT.DTERR) GO TO 7                                         0190
C                                                                       0191
C     IN THIS SECTION THE DEPENDENT VARIABLES ARE PRINTED OUT           0192
```

```
C                                                                       0193
      WRITE(6,116) TIME                                                 0194
      WRITE(6,160)                                                      0195
 160  FORMAT(24H THE VORTICITY FIELD IS  ///)                           0196
      WRITE(6,173) V                                                    0197
 173  FORMAT(6E14.5)                                                    0198
      WRITE(6,118)                                                      0199
      CALL GRAOUT(LV,LO,NOUT,V,NM,IA,JA,M2,N,BLANK)                     0200
      WRITE(6,116) TIME                                                 0201
 116  FORMAT(1H1,43H THE NONDIMENSIONAL TIME INTO THE MODEL IS ,E12.3//) 0202
      WRITE(6,117) ITER                                                 0203
 117  FORMAT(39H THE VALUE OF THE STREAM FUNCTION AFTER ,I8,            0204
     E       11H ITERATIONS   ///)                                      0205
      WRITE(6,173) S                                                    0206
      WRITE(6,118)                                                      0207
 118  FORMAT(1H1)                                                       0208
      CALL GRAOUT(LV,LO,NOUT,S,NM,IA,JA,M2,N,BLANK)                     0209
  7   CONTINUE                                                          0210
C                                                                       0211
CC    A MEASURE OF THE TOTAL KINETIC ENERGY OF THE FLOW                 0212
CC    IS MONITORED AS A STABILITY CHECK.                                0213
C                                                                       0214
      FAC = 0.0                                                         0215
      DO 152 K = 1,M2                                                   0216
      I = IA(K)                                                         0217
      J = JA(K)                                                         0218
      DO 151 L = I,J                                                    0219
      A1 = S(L)                                                         0220
      FAC = FAC+(S(L+1)-A1)**2+(S(L-N+1)-A1)**2+(S(L-N)-A1)**2          0221
     E   +(S(L-1)-A1)**2+(S(L+N-1)-A1)**2+(S(L+N)-A1)**2                0222
 151  CONTINUE                                                          0223
 152  CONTINUE                                                          0224
      WRITE(6,153) TIME,FAC                                             0225
 153  FORMAT(14H THE TIME IS =,F10.4,22H THE KINETIC ENERGY IS,E14.3)   0226
      WRITE(6,379) VORT                                                 0227
 379  FORMAT(25H THE TOTAL VORTICITY IS   ,E14.3)                       0228
      VORT = 0.0                                                        0229
      TIME = TIME + DT                                                  0230
      IF(TIME.GT.TMAX) GO TO 16                                         0231
C                                                                       0232
CC    THE LOCAL TIME CHANGE IN THE VORTICITY AT THE PRESENT             0233
CC    TIME STEP IS NOW CALCULATED FOR ALL INTERNAL POINTS               0234
C                                                                       0235
      DO 10 K = 1,M2                                                    0236
      I = IA(K)                                                         0237
      J = JA(K)                                                         0238
      DO 9 L = I,J                                                      0239
      L1 = L+1                                                          0240
```

```
      L2 = L-N+1                                                        0241
      L3 = L-N                                                          0242
      L4 = L-1                                                          0243
      L5 = L+N-1                                                        0244
      L6 = L+N                                                          0245
C                                                                       0246
CCC   HERE THE CONTRIBUTION FROM THE ADVECTION OF POTENTIAL             0247
CC    VORTICITY IS CALCULATED                                           0248
C                                                                       0249
      A1 = (C1*V(L1)+F(L1))/H(L1)                                       0250
      A2 = (C1*V(L2)+F(L2))/H(L2)                                       0251
      A3 = (C1*V(L3)+F(L3))/H(L3)                                       0252
      A4 = (C1*V(L4)+F(L4))/H(L4)                                       0253
      A5 = (C1*V(L5)+F(L5))/H(L5)                                       0254
      A6 = (C1*V(L6)+F(L6))/H(L6)                                       0255
      FAC =((S(L2)-S(L1))*(A2+A1)+(S(L3)-S(L2))*(A3+A2)                 0256
     E    +(S(L4)-S(L3))*(A4+A3)+(S(L5)-S(L4))*(A5+A4)+(S(L6)-S(L5))    0257
     E    *(A6+A5)+(S(L1)-S(L6))*(A1+A6))/(2.0*AREA)                    0258
C                                                                       0259
CCC   THE CONTRIBUTION FROM THE DIFFUSION OF VORTICITY IS NOW           0260
CCC   CALCULATED AND ADDED ON                                           0261
C                                                                       0262
      FAC = FAC+C2*(V(L1)+V(L2)+V(L3)+V(L4)+V(L5)+V(L6)-6.0*V(L))       0263
C                                                                       0264
CCC   THE CONTRIBUTION FROM THE CURL OF THE WIND IS ADDED ON            0265
C                                                                       0266
      FAC = FAC + T(L)                                                  0267
C                                                                       0268
CCC   THE CONTRIBUTION OF THE BOTTOM STRESS IS ADDED ON AND THE         0269
CCC   RESULT STORED IN G2(L)                                            0270
C                                                                       0271
      G2(L) = FAC - (C3/H(L)) * (V(L) + ((HC1(L) * (S(L1) - S(L4) + 0.5 0272
     1    * (S(L2) - S(L5) - S(L3) + S(L6)))) + (HC2(L) * (S(L2) - S(L5) 0273
     2    + S(L3) - S(L6)))))                                           0274
   9  CONTINUE                                                          0275
  10  CONTINUE                                                          0276
C                                                                       0277
CC    THE NEW BOUNDARY VALUES FOR THE VORTICITY ARE                     0278
CC    CALCULATED IN THIS SECTION                                        0279
CC    FIRST THE LOCAL CHANGE OF VORTICITY IS CALCULATED FOR ALL BOUNDARY 0280
CC    POINTS EXCEPT WHERE INFLOW OCCURS, THOSE VALUES DO NOT CHANGE     0281
CC    WITH TIME.                                                        0282
C                                                                       0283
      DO 8 I = 1,NB                                                     0284
      L = IBC(7,I)                                                      0285
      LA(1) = L+1                                                       0286
      LA(2) = L-N+1                                                     0287
      LA(3) = L-N                                                       0288
```

```
      LA(4) = L-1                                                          0289
      LA(5) = L+N-1                                                        0290
      LA(6) = L+N                                                          0291
      FAC = 0.0                                                            0292
      TRI = 1.0                                                            0293
      DO 84 J = 1,6                                                        0294
      K = J+1                                                              0295
      IF(K.EQ.7) K = 1                                                     0296
      M = J-1                                                              0297
      IF(M.EQ.0) M = 6                                                     0298
C                                                                          0299
C     HERE THE CONTRIBUTION FROM ADVECTION IS CALCULATED IF THE TRIANGLE   0300
C     IS INTERIOR TO THE MODEL                                             0301
C                                                                          0302
      IF(IBC(J,I).EQ.0.OR.IBC(K,I).EQ.0) GO TO 81                          0303
      L1 = LA(J)                                                           0304
      L2 = LA(K)                                                           0305
      FAC = FAC + (S(L2)-S(L1))*((C1*V(L2)+F(L2))/H(L2) +                  0306
     E       (C1*V(L1)+F(L1))/H(L1))/(2.0*AREA)                            0307
C                                                                          0308
C     HERE THE CONTRIBUTION FROM DIFFUSION PARALLEL THE BOUNDARY IS        0309
C     CALCULATED                                                           0310
C                                                                          0311
 81   IF(IBC(J,I).NE.3.AND.IBC(J,I).NE.4) GO TO 811                        0312
      L1 = LA(J)                                                           0313
      FAC = FAC + C2*(V(L1)-V(L))/2.0                                      0314
C                                                                          0315
C     HERE THE CONTRIBUTION OF DIFFUSION FROM THE INTERIOR IS              0316
C     CALCULATED                                                           0317
C                                                                          0318
 811  IF(IBC(J,I).NE.1.AND.IBC(J,I).NE.2) GO TO 812                        0319
      L1 = LA(J)                                                           0320
      FAC = FAC + C2*(V(L1)-V(L))                                          0321
 812  CONTINUE                                                             0322
C                                                                          0323
C     HERE THE DIFFUSION NORMAL TO THE BOUNDARY IS CALCULATED IF A NO      0324
C     SLIP CONDITION IS USED.  NOTE LENGTH FACTORS CANCEL OUT, NO EXTRA    0325
C     COEFFICIENTS ARE NEEDED.                                             0326
C                                                                          0327
      IF(IBC(J,I).NE.2) GO TO 82                                           0328
      IF(IBC(M,I).EQ.2.AND.IBC(K,I).EQ.2) GO TO 82                         0329
      IF(IBC(M,I).EQ.2) GO TO 813                                          0330
      L1 = LA(J)                                                           0331
      L2 = LA(M)                                                           0332
      FAC = FAC + C2*(V(L1)-0.5*(V(L2)+V(L)))                              0333
 813  IF(IBC(K,I).EQ.2) GO TO 82                                           0334
      L1 = LA(J)                                                           0335
      L2 = LA(K)                                                           0336
```

```
      FAC = FAC + C2*(V(L1)-0.5*(V(L2)+V(L)))                                  0337
C                                                                              0338
C     NOW THE POSSIBILITY OF VORTICITY BEING ADVECTED OUT OF THE MODEL         0339
C     IS TAKEN CARE OF.                                                        0340
C                                                                              0341
 82   IF(IBC(J,I).NE.4) GO TO 83                                               0342
      L1 = LA(J)                                                               0343
      FAC = FAC - ABS(S(L1)-S(L))*((C1*V(L1)+F(L1))/H(L1)+(C1*V(L)             0344
     1     +F(L))/H(L))*0.5/AREA                                               0345
C                                                                              0346
C     HERE THE NUMBER OF OUTSIDE POINTS ARE COUNTED FOR THE AREA               0347
C     CORRECTION                                                               0348
C                                                                              0349
 83   IF(IBC(J,I).EQ.0) TRI = TRI + 1.0                                        0350
 84   CONTINUE                                                                 0351
C                                                                              0352
C     NOW THE LOCAL CHANGE IS CORRECTED FOR AREA                               0353
C                                                                              0354
      G2(L) = FAC*6.0/(6.0-TRI)                                                0355
 8    CONTINUE                                                                 0356
C                                                                              0357
C     THE NEW BOUNDARY VALUE FOR THE VORTICITY IS CALCULATED                   0358
C     USING THE ADAMS-BASHFORTH SCHEME AND G2 IS TRANSFERED                    0359
C     TO G1                                                                    0360
C                                                                              0361
      DO 80 I = 1,NB                                                           0362
      L = IBC(7,I)                                                             0363
      V(L) = V(L) + DT*(1.5*G2(L)-0.5*G1(L))                                   0364
      TRI = 1.0                                                                0365
      DO 888 J = 1,6                                                           0366
      IF(IBC(J,I).EQ.0) TRI = TRI + 1.0                                        0367
 888  CONTINUE                                                                 0368
      VORT = VORT + V(L)*(6.0-TRI)/H(L)                                        0369
 80   G1(L) = G2(L)                                                            0370
C                                                                              0371
C     THE NEW VALUE OF THE VORTICITY IS CALCULATED AT ALL                      0372
C     INTERIOR POINTS USING THE ADAMS-BASHFORTH METHOD AND                     0373
C     THE PRESENT VALUE OF THE LOCAL CHANGE IS TRANSFERED                      0374
C     FROM G2 TO G1 TO BE USED IN THE NEXT TIME STEP                           0375
C                                                                              0376
      DO 12 K = 1,M2                                                           0377
      I = IA(K)                                                                0378
      J = JA(K)                                                                0379
      DO 11 L = I,J                                                            0380
      V(L) = V(L)+DT*(1.5*G2(L)-0.5*G1(L))                                     0381
      VORT = VORT + V(L)*6.0/H(L)                                              0382
 11   G1(L) = G2(L)                                                            0383
 12   CONTINUE                                                                 0384
```

```
C                                                                       0385
CC      THE NEW VALUE OF THE STREAM FUNCTION IS OBTAINED                0386
CC      VIA THE RELAXATION OF THE NEW VORTICITY FIELD.  A               0387
CC      SWEEP PROCEDURE IS STARTED AND 100 SWEEPS IS SET AS             0388
CC      THE UPPER LIMIT ON PASSES THROUGH THE GRID                      0389
C                                                                       0390
        ITER = 100                                                      0391
        DO 15 II = 1,100                                                0392
        ERR = 0.0                                                       0393
        DO 14 K = 1,M2                                                  0394
        I = IA(K)                                                       0395
        J = JA(K)                                                       0396
        DO 13 L = I,J                                                   0397
        L1 = L+1                                                        0398
        L2 = L-N+1                                                      0399
        L3 = L-N                                                        0400
        L4 = L-1                                                        0401
        L5 = L+N-1                                                      0402
        L6 = L+N                                                        0403
        HO = H(L)                                                       0404
        FAC = 1.33333*(S(L1)/(H(L1)+HO)+S(L2)/(H(L2)+HO)                0405
       E    +S(L3)/(H(L3)+HO)+S(L4)/(H(L4)+HO)+S(L5)/(H(L5)+HO)         0406
       E    +S(L6)/(H(L6)+HO)-S(L)*RES(L))-V(L)                         0407
        FAC = FAC/(1.33333*RES(L))                                      0408
        S(L) = S(L)+R*FAC                                               0409
        FAC = ABS(FAC)                                                  0410
        IF(FAC.GT.ERR) ERR= FAC                                         0411
 13     CONTINUE                                                        0412
 14     CONTINUE                                                        0413
C                                                                       0414
CC      THE CONVERGENCE IS NOW CHECKED AND IF THE MAX                   0415
CC      RESIDUE IS LESS THEN CON THE ITERATION LOOP IS  EXITED          0416
C                                                                       0417
        IF(ERR.GT.CON) GO TO 15                                         0418
        ITER = II                                                       0419
        GO TO 155                                                       0420
 15     CONTINUE                                                        0421
 155    GO TO 6                                                         0422
 16     CONTINUE                                                        0423
        IF(IPUNCH.EQ.0) GO TO 17                                        0424
        WRITE(7,115) S                                                  0425
        WRITE(7,115) V                                                  0426
        WRITE(7,115) G1                                                 0427
 17     CONTINUE                                                        0428
        STOP                                                            0429
        END                                                             0430
```

```
      SUBROUTINE GRAOUT(LV,LO,NOUT,A,NM,IA,JA,M2,N,BLANK)                0431
      DIMENSION LV(20),LO(NOUT),A(NM),IA(M2),JA(M2)                      0432
C                                                                        0433
C     THIS SUBROUTINE TAKES THE INTERIOR VALUES IN ARRAY A SCALES        0434
C     THEM, TRANSFORMS THEM TO AN ALPHA CODE AND PRINTS                  0435
C     THEM OUT IN A CARTESIAN GRID                                       0436
C                                                                        0437
      M = NM/N                                                           0438
      DO 1 K = 1,NOUT                                                    0439
    1 LO(K) = BLANK                                                      0440
      K = IA(1)                                                          0441
      AMAX = A(K)                                                        0442
      AMIN = AMAX                                                        0443
      DO 3 K = 1,M2                                                      0444
      I = IA(K)                                                          0445
      J = JA(K)                                                          0446
      DO 2 L = I,J                                                       0447
      IF(A(L).GT.AMAX) AMAX = A(L)                                       0448
      IF(A(L).LT.AMIN) AMIN = A(L)                                       0449
    2 CONTINUE                                                           0450
    3 CONTINUE                                                           0451
      ADIF = AMAX-AMIN                                                   0452
      IF(ADIF.EQ.0.0) GO TO 8                                            0453
      DO 5 K = 1,M2                                                      0454
      I = IA(K)                                                          0455
      J = JA(K)                                                          0456
      DO 4 L = I,J                                                       0457
      LL = 2*L-1+((L-1)/N)*(M-1)                                         0458
      NN = IFIX(19.999*(AMAX-A(L))/ADIF)                                 0459
    4 LO(LL) = LV(NN+1)                                                  0460
    5 CONTINUE                                                           0461
      WRITE(6,6) LO                                                      0462
C                                                                        0463
C     THIS FORMAT CARD MUST BE CHANGED FOR DIFFERENT NM VALUES           0464
C     THE COEFFICIENT BEFORE THE A2 SHOULD BE (2N-1) + (M-1)             0465
C                                                                        0466
    6 FORMAT(1H0,19A2)                                                   0467
      RETURN                                                             0468
    8 WRITE(6,9)                                                         0469
    9 FORMAT(47H THE ARRAY HAD A CONSTANT VALUE NO GRAPH GIVEN  )        0470
      RETURN                                                             0471
      END                                                                0472
```

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)*

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Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3 REPORT TITLE

The Development of a Homogeneous Numerical Ocean Model for the Arctic Ocean

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5 AUTHOR(S) *(First name, middle initial, last name)*

J. A. Galt

6. REPORT DATE
July 1972

7a. TOTAL NO. OF PAGES
59

7b. NO. OF REFS
21

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

NPS-58Gl72071A

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
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13. ABSTRACT

A numerical ocean model driven by surface stress and a source-sink distribution is developed for a homogeneous ocean. Non-linearities, lateral friction and bottom friction are included. The basin shape can be varied to accommodate a large variety of configurations. Variable bathymetry and sources/sinks around the perimeter are included. The numerical scheme is conditionally stable and has second order accuracy in space and time.

A number of test cases are run to explore the dynamic significances of the various processes represented. The possible influence of these processes on the circulation of the Arctic ocean are discussed.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ocean Circulation Numerical Modeling Arctic Ocean						